

# **BSSRDF (Bidirectional Surface Scattering Distribution Function)**

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# BSSRDF

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## Abstract

*Traditional lightning models used for rendering images typically assume that the light exits at the same position that it goes in. This is not true especially in case of translucent materials. This paper introduces an efficient BSSRDF (bidirectional surface scattering distribution function) model developed by Henrik Wann Jensen that takes this physical phenomenon into account. Paper explains in detail how BSSRDF model is incorporated to standard Monte Carlo path tracing algorithm and gives intuitive explanations to complex mathematical formulas caused by physical phenomenon.*

## 1 INTRODUCTION

This paper concentrates on subsurface light transport that happens on translucent materials. Translucency is a material phenomenon where light travels through an object's surface rather than simply bouncing off the surface. Most non-metal surfaces contain a certain degree of translucency. A good example of translucent material is wax, see figure 1.

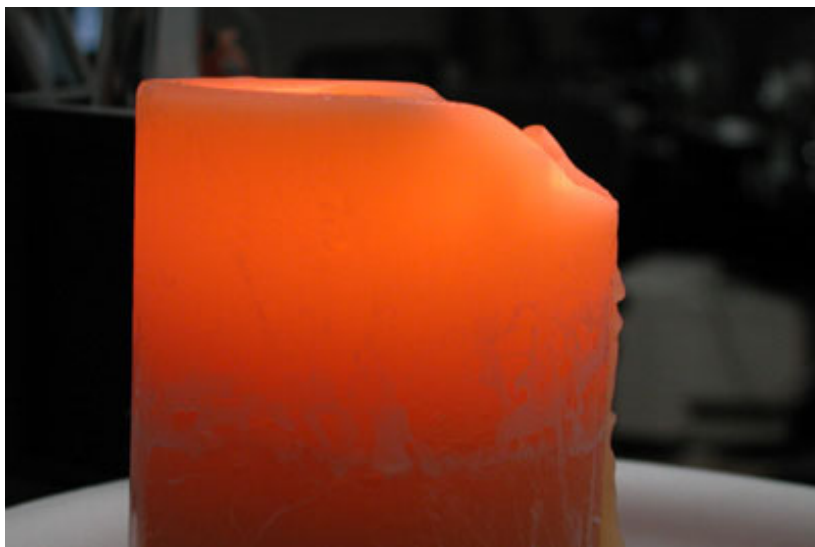


Figure 1. An example of translucent material, a glowing candle ([http://www.neilblevins.com/cg\\_education/tut28/tut28.htm](http://www.neilblevins.com/cg_education/tut28/tut28.htm) 1.4.2002).

When light encounters an obstacle on its path light is either absorbed or scattered. Obstacle can be a surface of different material or medium. It can also be a small particle or a molecule in a volumetric medium such as piece of fog or flame. The calculations needed for generating the illumination of light in volumetric medium become computationally heavy. Thereby it is not efficient to use a full-blown volume model for translucent surfaces even though subsurface scattering is a volumetric phenomenon. BSSRDF model tackles this problem, relying strongly on the theory of light transport in volumetric medium.

A simple method for calculating the scattering of light on a surface is to assume the surface to be a Lambertian, or in other words an ideal diffuse reflector. With Lambertian surfaces the reflected direction is assumed perfectly random and the radiance constant in all directions. See figure 2 (a). Quite much research has focused on developing models for the more general, bidirectional reflectance distribution function (BRDF). A surface's BRDF specifies how much of the light incident from any one direction is emitted in any second direction. BRDF model, originally introduced by Nicodemus & al. (1977), assumes that light striking at surface location is reflected at the same surface location. See figure 2 (b). BRDFs can be simple, such as the Lambertian model that causes a constant BRDF or complex, such as Cook-Torrance model. Assuming the light to exit at the same location as it entered works well for most materials, but not for translucent materials. See figure 3.

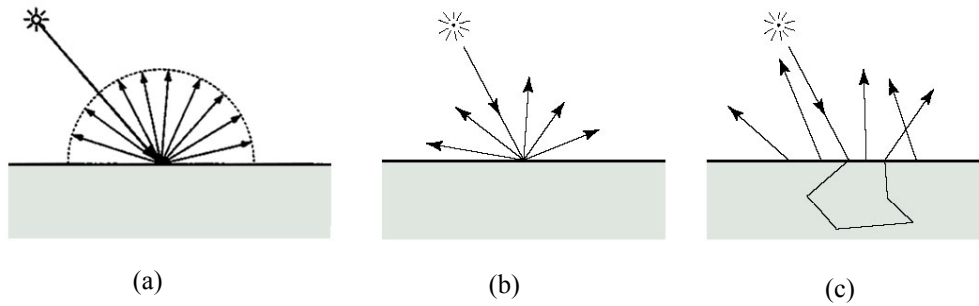


Figure 2. Scattering of light in (a) a Lambertian surface (b) a BRDF (c) BSSRDF(Jensen 2001a).



Figure 3. Image of a human face rendered with (a) BRDF (b) BSSRDF model (<http://graphics.stanford.edu/~henrik/> 1.4.2002).

When a beam of light is scattered by a material it normally enters the material and then scatters around before leaving the surface at a *different* location. See figures 2 (c) and 4. Bidirectional surface scattering distribution function (BSSRDF) developed by Jensen (2001a) describes this behavior correctly without assuming the light beam to leave the surface at a same location that it went in. Actually BRDF approximation is just a special case of BSSRDF (Jensen 2001a).

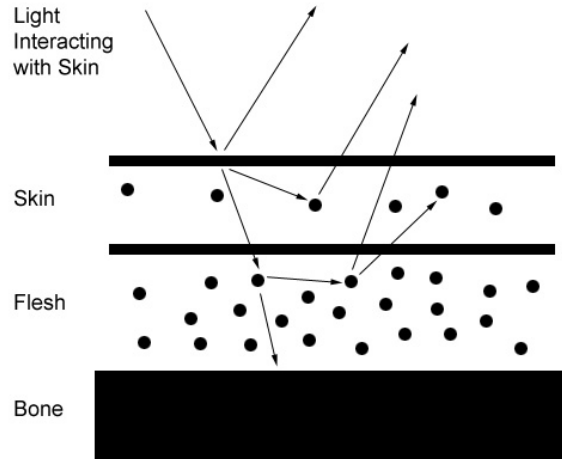


Figure 4. Light interaction with skin

([http://www.neilblevins.com/cg\\_education/tut28/tut28.htm](http://www.neilblevins.com/cg_education/tut28/tut28.htm) 1.4.2002).

Chapter 2 introduces the basic principles and equations of light scattering in general participating media (volume). Chapter 3 covers BSSRDF and describes its multi and single scattering components. Chapter 4 is dedicated to rendering BSSRDF with Monte Carlo path tracing.

## 2 LIGHT SCATTERING IN PARTICIPATING MEDIA

To perform a full simulation of subsurface scattering, it is necessary to solve the same basic equations as in the case of general participating media. However, in case of subsurface scattering quite many simplifications can be made. When a photon enters a participating media it can either continue unaffected through the medium or it can interact at a given location. When a photon interacts with a medium one of two things can happen: photon gets either scattered or absorbed. The probability of a photon being either scattered or absorbed as it moves through the medium is given by the scattering coefficient,  $\sigma_s$ , and the absorption coefficient  $\sigma_a$ .

The combined loss in radiance  $L$  in the direction  $\omega$  due to both out-scattering and absorption is (Jensen 2001b):

$$(\bar{\omega} \cdot \nabla)L(x, \bar{\omega}) = -\sigma_t(x)L(x, \bar{\omega}) \quad (1)$$

where  $\sigma_t = \sigma_s + \sigma_a$ .

As we move through the media there will also be a gain in radiance due to in scattering of light. This is given by following equation:

$$(\bar{\omega} \cdot \nabla)L(x, \bar{\omega}) = \sigma_s(x) \int_{\Omega_{4\pi}} p(x, \bar{\omega}', \bar{\omega}) L_i(x, \bar{\omega}') d\bar{\omega}' \quad (2)$$

where the incident radiance  $L_i$  is integrated over all directions of the sphere  $\Omega_{4\pi}$ . Phase function  $p(x, \bar{\omega}', \bar{\omega})$  is a material property that describes the distribution of the scattered light. Quite often the phase function depends only on the angle  $\theta$  between the incoming ray  $\bar{\omega}'$  and the scattered ray  $\bar{\omega}$ , and can be written as  $p(\theta)$  (Jensen 2001b, p. 115). Figure 5 illustrates Henyey-Greenstein phase functions  $p(\theta)$ . In case of isotropic scattering the photon is scattered in random direction without a history where it came from. For isotropic scattering the phase function is constant and looks like figure 5 (a). A useful property calculated of phase function is the mean cosine,  $g$ , of scattering direction:

$$g(x) = \int_{\Omega_{4\pi}} p(x, \bar{\omega}', \bar{\omega}) \cos \theta d\bar{\omega}'$$

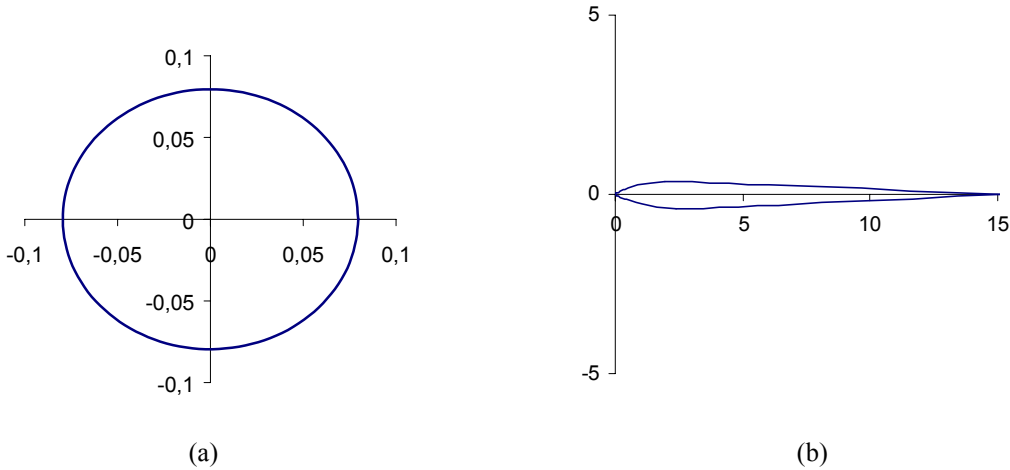


Figure 5. The Henyey-Greenstein phase function (a)  $g = 0$ , (b)  $g = 0.9$ .

There also can be a gain in radiance due to emission  $L_e$  from the medium, e.g. because of flames:

$$(\bar{\omega} \cdot \nabla)L(x, \bar{\omega}) = \sigma_a L_e(x, \bar{\omega}) \quad (3)$$

By combining equations (1), (2) and (3) it is possible to find the total change in radiance per unit distance. After integrating the both sides of combined equation for a segment of length  $s$  the commonly known *volume rendering equation* can be formed:

$$\begin{aligned}
L(x, \bar{\omega}) = & \int_0^s e^{-\tau(x, x')} \sigma_a(x') L_e(x') dx' + \\
& \int_0^s e^{-\tau(x, x')} \sigma_s(x') \int_{\Omega_{4\pi}} p(x', \bar{\omega}', \bar{\omega}) L_i(x', \bar{\omega}') d\bar{\omega}' dx' + \\
& e^{-\tau(x, x+s\bar{\omega})} L(x+s\bar{\omega}, \bar{\omega})
\end{aligned} \tag{4}$$

where the optical depth  $\tau(x, x')$  is given by:

$$\tau(x, x') = \int_x^{x'} \sigma_t(t) dt \tag{5}$$

Analytical solutions to volume rendering equation can found only on very few cases. Hence in general, volume rendering equation has to be solved numerically. A common scheme for solving numerically volume rendering equation is ray-marching, see details in Jensen (2001b, p. 119-121).

### 3 SURFACE SCATTERING

The most advanced and accurate, yet computationally feasible model for calculating subsurface scattering is the BSSRDF method introduced by Jensen (2001a). This method relies on the observation that the scattering of light is formed of two components: *single* and *multiple scattering*. Physically single scattering is interpreted as illumination from a single scattering event and multiple scattering as illumination due to integer number of scattering events. The solution for single scattering will be exact while multiple scattering is solved with a dipole point source approximation. For example, human skin's multiple scattering term is very high, whereas its single scattering term is very small. A substance such as marble or wax has a much larger contribution from the single scattering effect, and consequently less contribution from the diffusion (multiple) component. Figure 6 illustrates single and multiple scattering components in case of a leaf. The leaf is illuminated from behind with a laser pointer. The strong, focused center, which is the laser entering one side of the leaf and exiting through the other, much like light travels through a piece of glass and comes out at a different angle due to refraction, is the single scattering component. The soft glow surrounding the leaf is the multiscattering component.



Figure 6. Single and multiple scattering  
[http://www.neilblevins.com/cg\\_education/tut28/tut28.htm](http://www.neilblevins.com/cg_education/tut28/tut28.htm) 1.4.2002).

The BSSRDF,  $S$ , relates the differential reflected (outgoing) radiance  $dL_r(x, \omega)$  at the point  $x$  in direction  $\omega$ , to the incident flux  $\Phi_i(x', \omega')$  at the point  $x'$  from direction  $\omega'$  (Jensen 2001a):

$$S(x', \omega', x, \omega) = \frac{dL_r(x, \omega)}{d\Phi_i(x', \omega')} \quad (6)$$

As can be seen  $S$  is a function of both incoming position and direction as well as the outgoing position and direction. As mentioned BSSRDF consists of two components. Hence following is true:

$$S = S^{(l)} + S_d \quad (7)$$

where  $S^{(l)}$  is the single scattering and  $S_d$  is the diffuse approximation term.

### 3.1 Diffuse approximation

The formulation of diffuse approximation is based on the observation that the light distribution in highly scattering media tends to be isotropic. This is true even if the initial light source and phase function are highly anisotropic (Jensen 2001a). Hence in diffuse approximation the phase function  $p(x, \omega', \omega)$  is constant. In this situation the

radiance  $L$  can be approximated by a two-term expansion:

$$L(x, \bar{\omega}) = \frac{1}{4\pi} \phi(x) + \frac{3}{4\pi} \bar{\omega} \cdot \bar{E}(x) \quad (8)$$

where scalar and vector irradiances  $\phi(x)$  and  $\bar{E}(x)$ , that describe the exiting radiant flux per differential area are:

$$\begin{cases} \phi(x) = \int_{\Omega_{4\pi}} L(x, \bar{\omega}) d\omega \\ \bar{E}(x) = \int_{\Omega_{4\pi}} L(x, \bar{\omega}) \bar{\omega} d\omega \end{cases}$$

By combining equation (8) and the volume rendering equation (4) it is possible to form the classic diffusion equation (for details see Jensen (2001a)):

$$D\nabla^2 \phi(x) = \sigma_a \phi(x) - Q_0(x) + 3D\bar{\nabla} \cdot \bar{Q}_1(x) \quad (9)$$

where  $Q_0$  and  $\bar{Q}_1$  are 0<sup>th</sup> and 1<sup>st</sup> order source terms. The diffusion constant  $D$  is defined in terms of scattering and absorption coefficients,  $\sigma_s$  and  $\sigma_a$ , and the mean cosine,  $g$ , of scattering angle:

$$\begin{cases} D = \frac{1}{3\sigma_t'} \\ \sigma_t' = \sigma_s' + \sigma_a = \sigma_s(1 - g) + \sigma_a \end{cases}$$

Diffusion equation (9) needs to be solved for exiting scalar irradiance  $\phi$  in order to find the diffusion term of BSSRDF. There is an exact, analytical solution for diffusion equation in case of an infinite medium. However in the current case, which is the case of finite medium the diffusion equation does not in general have an analytical solution. Jensen (2001a) tackled the problem by using a method introduced by Eason & al. (1972) and Farrell & al. (1992). In this method the incoming ray is transformed into a dipole source. That is, two light sources are positioned near the surface in such way that the required boundary conditions are met. One, positive, real light source is located beneath the surface at the distance  $z_r$  and the other, negative, virtual light source above it at the distance  $z_v$ . See figure 7.



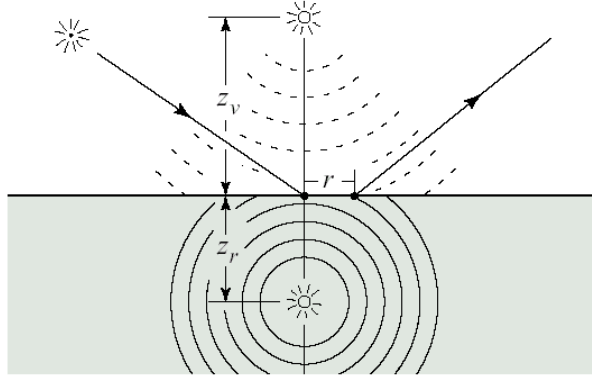


Figure 7. Dipole source model for solving the diffuse approximation (Jensen 2001a).

Jensen (2001a) proposes to position the real and virtual light sources straight below and above  $x'$  at following distances:

$$\begin{cases} z_r = \frac{1}{\sigma'_t} \\ z_v = z_r + 4AD = z_r + 4D \frac{1 + F_{dr}}{1 - F_{dr}} \end{cases}$$

where

$$F_{dr} = -\frac{1.440}{\eta^2} + \frac{0.710}{\eta} + 0.668 + 0.0636\eta$$

$F_{dr}$  represents a rational approximation of diffuse reflectance.  $\eta$  in  $F_{dr}$  is the relative index of refraction between the two mediums. The resulting solution to diffusion equation for scalar irradiance is:

$$\phi(x) = \frac{\Phi}{4\pi D} \left( \frac{e^{-\sigma_{tr}d_r}}{d_r} - \frac{e^{-\sigma_{tr}d_v}}{d_v} \right) \quad (10)$$

where  $d_r$  is the distance from  $x$  to the real light source and  $d_v$  from  $x$  to the virtual light source:

$$\begin{cases} \sigma_{tr} = \sqrt{3\sigma_a\sigma'_t} \\ d_r = \|x - x_r\| \\ d_v = \|x - x_v\| \end{cases}$$

The diffuse contribution of BSSRDF due to subsurface scattering is equal to the radiant

exitance (flux leaving the surface) divided by the incident flux (Jensen 2001a):

$$\begin{aligned}
 R_d(\|x'-x\|) &= -D \frac{\bar{n} \cdot \bar{\nabla} \phi(x_s)}{d\Phi_i} \\
 &= \frac{\sigma'_s / \sigma_a}{4\pi} \left[ (\sigma_{tr} d_r + 1) \frac{e^{-\sigma_r d_r}}{\sigma'_t d_r^3} + z_v (\sigma_{tr} d_r + 1) \frac{e^{-\sigma_r d_v}}{\sigma'_t d_v^3} \right]
 \end{aligned} \tag{11}$$

However  $R_d$  does not contain the contribution from direct reflection of the surface (specular term). Specular reflection is taken into account with Fresnel's model (for details see Jensen (2001b, p. 23-24)). The Fresnel reflection needs to be taken into account at the boundary of both incoming light and the outgoing radiance (Jensen 2001a). Hence the final multiple scattering component  $S_d$  of the BSSRDF is:

$$S_d(x', \bar{\omega}', x, \bar{\omega}) = \frac{1}{\pi} F_t(\eta, \bar{\omega}') R_d(\|x'-x\|) F_t(\eta, \bar{\omega}) \tag{12}$$

Chapter 4 explains the usage of equation (12) in practice.

### 3.2 Single Scattering

Physically single scattering can be interpreted as illumination from a single scattering event. Mathematically this means the first order solution to volume rendering equation (4). Single scattering occurs only when the refracted incoming and outgoing rays intersect as shown in figure 8. According to Jensen (2001a) the total outgoing radiance  $L^{(1)}$  due to single scattering is computed as an integral over path length  $s$  along the refracted outgoing ray:

$$\begin{aligned}
 L^{(1)}(x, \bar{\omega}) &= \sigma_s(x) \int_{\Omega_{2\pi}} Fp(x', \bar{\omega}', \bar{\omega}) \int_0^\infty e^{\sigma_{tc}s} L_i(x', \bar{\omega}') ds d\bar{\omega}' = \\
 &\int_A \int_{\Omega_{2\pi}} S^{(1)}(x', \bar{\omega}', x, \bar{\omega}) L_i(x', \bar{\omega}') (\bar{n} \cdot \bar{\omega}') d\omega' dA(x')
 \end{aligned} \tag{13}$$

where

$$\begin{cases} F = F_t(\eta, \bar{\omega}') F_t(\eta, \bar{\omega}) \\ \sigma_{tc} = \sigma_t(x) + G\sigma_t(x') \end{cases} \tag{14}$$

$G$  is an a geometry factor and for flat surface its value is:

$$G = \frac{|\bar{n}' \cdot \bar{\omega}|}{|\bar{n}' \cdot \bar{\omega}'|} \tag{15}$$

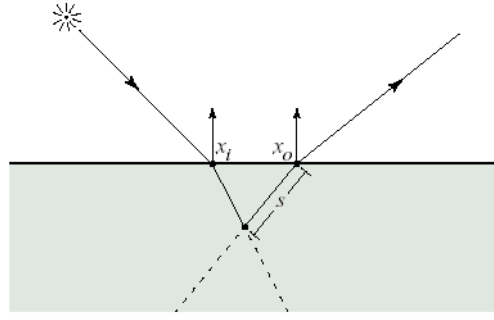


Figure 8. Single scattering illustration (Jensen 2001a).

In practice equation (13) has only theoretical meaning since the second and third lines of the equation define implicitly the single scattering component  $S^{(1)}$  of BSSRDF. Chapter 4 and explains intuitively the formation the practical equation for single scattering term used in rendering instead of equation (13).

#### 4 RENDERING USING MONTE CARLO PATH TRACING AND BSSRDF

Calculating the illumination using BSSRDF requires evaluation of integral equations, as seen in previous sections. These integration problems can be handled by tracing a random ray within the integration domain multiple times and then averaging the result to estimate the value of integral. This concept is generally known as *Monte Carlo integration*. There is a lot of literature that explain the Monte Carlo concept, for basics see for example Hearn and Baker (1997, p. 624). When Monte Carlo integration is incorporated to standard ray tracing algorithm the well-known path tracing algorithm is formed. Jensen (2001b, p. 33-50) covers nicely the standard ray and path tracing algorithms. In this paper the basics of well-known tracing algorithms are not covered, instead the incorporation of BSSRDF to path tracing is explained in detail.

There is a general difference in BSSRDF path tracing algorithm compared to path tracing algorithms used with traditional lightning models (based on BRDFs). That is, in case of traditional models at each ray-object intersection only the direction of a shadow ray is sampled, while in case BSSRDF both the direction and the start point of a shadow ray are sampled. See figure 9.

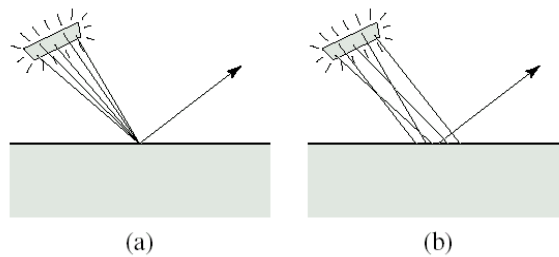


Figure 9. (a) Sampling with traditional lightning model. Only direction is sampled. (b) Sampling with BSSRDF. Both direction and start point are sampled. (Jensen 2001a)

It is convenient to separate the sampling of diffuse and single scattering terms since, diffuse term samples are distributed around the exit point  $x$  and single scattering term samples must be along the refracted outgoing ray.

Radiance  $L_d$  due to **multiple scattering** is evaluated using BSSRDF  $S_d$  defined by equation (12). Jensen (2001a) proposes diffuse term to be multiplied by exponential falloff. In order to obtain the outgoing radiance  $S_d$  needs to be integrated over the surface area and over all incoming directions. Because of Monte Carlo scheme the  $S_d$  is constant at one sample event. Incident illumination is assumed to be constant at one sample event as well. Hence the radiance due to multiple scattering becomes following:

$$L_d(x, \bar{\omega}) = \pi \frac{d^2}{4} S_d(d) L'(x', \bar{\omega}') \sigma_r e^{-\sigma_r d} \quad (16)$$

where  $\pi d^2/4$  is due to the integration over the surface.  $d$  is a random distance measured from  $x$  that is sampled for every ray.

When using Monte Carlo path tracing radiance  $L^{(l)}$  due to **single scattering** event can be calculated using following equation:

$$L^{(1)}(x, \bar{\omega}) = \frac{\sigma_s(x) Fp(\bar{\omega}', \bar{\omega})}{\sigma_{tc}} e^{-s_i' \sigma_i(x')} e^{-s_o' \sigma_i(x)} L'(x', \bar{\omega}') \quad (17)$$

where  $s_o'$  is a sampled random distance along the primary ray and  $s_i'$  is the distance along the shadow ray as illustrated in figure 10:

$$\begin{cases} s_o' = \log(\xi) / \sigma_i(x) \\ s_i' = s_i \frac{|\bar{\omega}' \cdot \bar{n}|}{\sqrt{1 - \left(\frac{1}{\eta}\right)^2 (1 - |\bar{\omega}' \cdot \bar{n}|^2)}} \end{cases} \quad (18)$$

$\xi$  is a uniformly distributed number between 0 and 1.  $s_i'$  is an approximation based on the Snell's law for the distance that incoming sample ray moves through the material. An approximation is used because for arbitrary geometry it is difficult to find the point where the shadow ray is refracted.

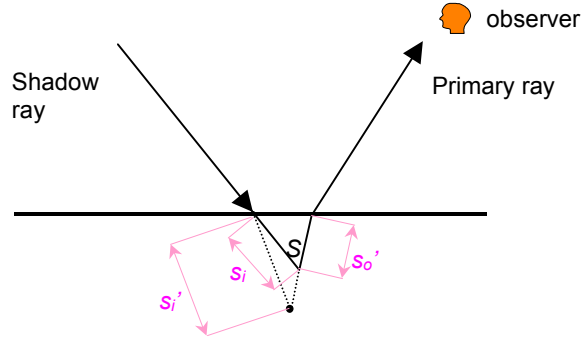


Figure 10. Illustration of the distances used in the evaluation of single scattering.

Intuitively equation (17) can be interpreted as follows. The incoming radiance  $L'$  is attenuated twice. The first attenuation happens when the ray comes in and moves through the material (distance  $s_i'$ ). The second attenuation happens after the scattering event when the ray moves through the material to get out (distance  $s_o'$ ). Exponential terms in equation (17) describe these attenuations. The division term in front of the exponential terms consists of a Fresnel term and the phase function. Those describe in what direction the scattering happens.

The BSSRDF path tracing algorithm that takes one shadow ray into account, as suggested in Jensen (2001b, p. 39), is described below as pseudo code. Note that figure 10 supports the algorithm description.

```
render image using path tracing
  for each pixel
    global color = 0
    for each sample
      pick ray from observer through random position in pixel
      global color = global color + trace(ray)
    pixel color = global color/#samples

trace(ray)
  find nearest intersection point with scene
  global color = shade single scattering(point)
  global color = color + shade multiple scattering(point)
  return global color
```

```

shade single scattering(point)
    color = 0
    generate random sample distance  $s_0'$ 
    calculate point  $S$  for refracted primary ray
    color = trace intersection color with other object (randomly
        reflected shadow ray from point  $S$ )
    calculate shadow ray - surface intersection point
    calculate distance  $s_i'$ 
    color = calculate single scattering(color,  $s_0'$ ,  $s_i'$ )
    return color

shade multiple scattering(point)
    color = 0
    for each light source
        test visibility of random position on light source
        if visible
            color = color + light source color
    color = color + trace intersection color with other
        objects (randomly reflected shadow ray)
    generate random sample distance  $d$ 
    color = calculate multiple scattering(color,  $d$ )
    return color

```

To get a decent accuracy for the Monte Carlo integration enough samples need to be used. If too few samples are used the final image becomes noisy, see figure 11. Typically the simulation is carried out with at least 100 samples per pixel (Jensen 2001b, p. 30).



Figure 11. Too few samples can be seen as noise in the final rendering (<http://www.cc.gatech.edu/gvu/people/Phd/Charles.Patterson/research/gsii/gsii.html> 14.4.2002).

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